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Resonance decays and partial coherence in Bose-Einstein correlations

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We investigate the influence of resonances on Bose-Einstein correlations (BEC's) in the presence of coherence of the pion field. For this purpose a realistic description of resonance production via hydrodynamics is attempted by constraining the initial and freeze-out conditions so that the single inclusive rapidity and transverse momentum distributions are correctly reproduced. We find that even a totally coherent source of "direct" pions leads to an appreciable apparent chaoticity because of the important role played by resonances in distorting the correlation function. On the other hand, kaons, being much less affected by resonance decays, seem to be ideal candidates for the experimental investigation of coherence in BEC's. Quantitative predictions of BEC's in $S+S$ reactions at 200A GeV are made.

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I. INTRODUCTION

The majority of secondaries produced in high-energy collisions are pions. A large fraction (between 40% and 80%) of these pions arise from resonances [1]. Since the resonances have finite lifetimes and momenta, their decay products are created in general outside the production region of the "direct" pions (i.e., pions produced directly from the source) and resonances. As a consequence, the two-particle correlation function of pions reflects not only the geometry of the (primary) source but also the momentum spectra and lifetimes of resonances [2]. Kaons are much less affected by this circumstance [3]; however, correlation experiments with kaons are much more difficult because of the low statistics.

The distortion of the two-particle correlation function due to resonance decay was studied, among other things, in Refs. [4–12]. Two obvious effects emerge: (a) The effective radius of the source increases; i.e., the width of the correlation function decreases. (b) Because of the finite experimental resolution in the momentum difference, the presence of very-long-lived resonances leads to an apparent decrease of the intercept of the correlation function to values below 2. Effect (b) is particularly important if one wants to draw conclusions from the intercept about a possible contribution of a coherent component in multiparticle production.

In the present paper, we continue the investigation of the influence of resonances on the two-particle correlation function by studying for the first time this problem within a realistic hydrodynamical approach and in the presence of a coherent pion field. The hydrodynamial approach appears for several reasons to be a good candidate for this problem: (i) Only through hydrodynamics, information about initial conditions, freeze-out, and the equation of state can be obtained; (ii) it provides also the single inclusive distributions which are intimately connected with higher-order distributions; (iii) it provides both the weights as well as the space-time and momentum distributions of resonances.

A comparison of pion and kaon Bose-Einstein correlations (BEC's) in heavy-ion reactions was made so far only in [3] and even there only for chaotic sources. On the other hand, in [3] a superposition of noninteracting strings is assumed which can be considered as a barebone model of nuclear collisions in which collective nuclear effects are absent. The presence of collective nuclear effects, however, appears almost inavoidable [13], and to take this into account in the string model scenario would necessitate the introduction of an interaction between strings [14]. Furthermore, some cross sections for resonance productions, which are used in the Lund model and which enter in the string model, are apparently in disagreement with data [15] and it is unclear how this affects the conclusion of Ref. [3].

In particular, we discuss the behavior of the correlation at small momentum differences. All the relevant effects are taken into account: resonance decay, longitudinal as well as transverse expansion, and modifications due to the presence of partial coherence and particle misidentification. For illustration, we apply the formalism to a full three-dimensional solution of the relativistic hydrodynamic equations, which describes the experimentally observed rapidity and transverse momentum distributions of mesons and protons, and compute the two-particle correlation functions for $S+S$ at 200A GeV.

In Sec. II we present the basic formalism and show how the experimentally observed intercept of the correlation function is influenced by the interplay between contributions from the decay of long-lived resonances and

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partial coherence. Section III contains the results of the numerical calculations for the concrete case of a relativistic nucleus-nucleus collision. The effects of resonance decays are illustrated through the comparison of pion and kaon interferometry, and the effects of the contribution of long-lived resonances as well as partial coherence on the apparent intercept of the correlation function are demonstrated. Finally, a summary of the main results and conclusions are presented in Sec. IV.

II. GENERAL FORMALISM

A. Contribution of resonance decays to Bose-Einstein correlations

For the case of a purely chaotic source, the formalism to take into account the effects of resonance decays on the Bose-Einstein correlation function can be found, e.g., in [7,9,10]. Below, we present a straightforward extension of the approach employed in [16] which is based on the Wigner function formulation of Ref. [17].

The correlation function of two identical particles of momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ can be written as

$$C_2(\mathbf{p}_1,\mathbf{p}_2)=1+\frac{A_{12}A_{21}}{A_{11}A_{22}}, \tag{1}$$

where the matrix elements A_{ij} are given in terms of source functions $g(x,k)$ as

$$A_{ij}=\sqrt{E_iE_j}\langle a^\dagger(\mathbf{p}_i)a(\mathbf{p}_j)\rangle=\int d^4x g(x_\mu,k^\mu)e^{iq^\mu x_\mu}. \tag{2}$$

Here, $a^\dagger(\mathbf{p})$ and $a(\mathbf{p})$ are the creation operator and the annihilation operator of a particle of momentum $\mathbf{p}$, and the four-momenta $k^\mu=\frac{1}{2}(p_i^\mu+p_j^\mu)$ and $q^\mu=p_i^\mu-p_j^\mu$ are the average momentum and the relative momentum of the particle pair, respectively. The interpretation of the source function $g(x_\mu,p^\mu)$ as the quantum analogue of the mean number of particles of momentum p^μ at the space-time point x_μ enables us to decompose g with respect to the origin of the produced hadrons. For instance, if the particles under consideration are two identical pions (e.g., two π^-), one has

$$g(x_\mu,p^\mu)=g_\pi^{\mathrm{dir}}(x_\mu,p^\mu)+\sum_{\mathrm{res}=\rho,\omega,\eta,\ldots} g_{\mathrm{res}\to\pi}(x_\mu,p^\mu), \tag{3}$$

where the labels dir and res$\to\pi$ refer to direct pions and to pions which are produced through the decay of resonances (such as ρ, ω, η, etc.), respectively. The contribution from a resonance decay $g_{\mathrm{res}\to\pi}(x_\mu,p^\mu)$ can be expressed in terms of the source function of that resonance itself, $g_{\mathrm{res}}^{\mathrm{dir}}(x_\mu^*,p^{*\mu})$, as follows (from here on, quantities related to a resonance will be labeled with a superscript star). Consider a resonance of width Γ, which is created at a space-time point x_μ^* and after a proper time τ decays into $\pi+X$ at $x^\mu=x^{*\mu}+(\tau/m^*)p^{*\mu}$. As the fluctuations of the decay time τ are described by the probability distribution $\Gamma\exp(-\Gamma\tau)$, one obtains

$$g_{\mathrm{res}\to\pi}(x_\mu,p^\mu)=\int\frac{d^3p^*}{E^*}\int d^4x^*\int_0^\infty d\tau\,\Gamma\exp(-\Gamma\tau)\delta^4\left[x^\mu-\left[x^{*\mu}+\frac{\tau}{m^*}p^{*\mu}\right]\right]\Phi_{\mathrm{res}\to\pi}(p^{*\mu},p^\mu)g_{\mathrm{res}}^{\mathrm{dir}}(x_\mu^*,p^{*\mu}), \tag{4}$$

where $\Phi_{\mathrm{res}\to\pi}(p^{*\mu},p^\mu)$ describes the probability for a resonance of momentum $p^{*\mu}$ to produce a pion of momentum p^μ.

We shall assume that the decay is governed by phase space. For an isotropic two-body decay, this implies

$$\Phi_{\mathrm{res}\to\pi}(p^{*\mu},p^\mu)=\frac{b}{4\pi p_0}\delta\left[E_0-\frac{p_\mu^*p^\mu}{m^*}\right], \tag{5}$$

where b is the branching ratio, and

$$E_0=\frac{1}{2m^*}(m^{*2}+m_\pi^2-m_X^2),$$

$$p_0=\frac{1}{2m^*}\sqrt{[m^{*2}-(m_\pi+m_X)^2][m^{*2}-(m_\pi-m_X)^2]}.$$

Three-body decays are treated in an analogous way [18].

Equations (1)–(6) allow us to calculate the two-particle correlation function provided the source distributions for the direct production of pions and resonances are known. In a hydrodynamic approach, these are [16]

$$g_\alpha^{\mathrm{dir}}(x_\mu,p^\mu)=\frac{2J+1}{(2\pi)^3}\int_\Sigma\frac{p^\mu d\sigma_\mu(x'_\mu)\,\delta^4(x_\mu-x'_\mu)}{\exp\left[\dfrac{p^\mu u_\mu(x'_\mu)-B\mu_B(x'_\mu)-S\mu_S(x'_\mu)}{T_f(x'_\mu)}\right]\pm1}, \tag{6}$$

where $d\sigma^\mu$ is the differential volume element and the integration is performed over the freeze-out hypersurface Σ. $u^\mu(x)$ and T_f are the four-velocity of the fluid element at point x and the freeze-out temperature, respectively. B and S are the baryon number and the strangeness of the particle species labeled α, respectively, and μ_B and μ_S are the corresponding chemical potentials. J is the spin of the particle. An evaluation of the integrals in Eqs. (2) and (4) is performed in Appendix A.

B. Partial coherence

In the previous subsection, it has been assumed that the particle source is completely chaotic [indeed, Eqs. (1) and (2) would imply that the intercept of the Bose-Einstein correlation function $C_2(\mathbf{p},\mathbf{p})$ must always be equal to 2]. However, a more general quantum statistical treatment also has to take into account the possibility of the contribution of a coherent component to the multiparticle production process.

In this section, we will consider the effects of introducing an additional coherent (nonfluctuating) field component into the formalism described above. We shall restrict ourselves to the case of additive coherence, i.e., of a superposition of a chaotic and a coherent component in the pion field. This approach is based on general rules of quantum statistics. For other treatments of coherence, cf. e.g., [19–21]. We are not concerned here with an explanation of the origin of partial coherence. Rather, we shall assume that a fraction of the secondaries is emitted coherently, and study the resultant behavior of the two-particle correlation function.

At this point, the question arises of whether or not pions which are created via resonance decays can exhibit partial coherence—a question which cannot be decided in the absence of a definite model. In the following, it will be assumed that only the directly produced pions contain a coherent component. This assumption is conservative since it underestimates the amount of coherence. In this case the fraction of directly produced pions which is emitted chaotically, i.e., $p_{\rm dir}$, is the "true" chaoticity $(0<p_{\rm dir}<1)$. In general [22], both the chaoticity parameter and the phase of the coherent field component will depend on the momentum of a particle, the momentum dependence depending on the particular model under consideration. Below, for the sake of simplicity, this momentum dependence will be neglected; i.e., it will be assumed that the chaoticity and the phase of the coherent component are approximately constant over the momentum region defined by the range of the Bose-Einstein correlations (that is to say, a momentum range characterized by the width of the correlation function).

To be specific, we decompose the matrix element A_{ij} into a direct and a resonance contribution [cf. Eq. (3)]:

$$A_{ij}=A_{ij}^{\pi}+\sum_{{\rm res}=\rho,\omega,\eta,\ldots} A_{ij}^{\rm res} , \tag{7}$$

where we have used the obvious notation

$$A_{ij}^{\pi}=\int d^4x\, g_{\pi}^{\rm dir}(x_\mu,k^\mu)e^{iq^\mu x_\mu} , \tag{8}$$

$$A_{ij}^{\rm res}=\int d^4x\, g_{{\rm res}\to\pi}(x_\mu,k^\mu)e^{iq^\mu x_\mu} , \tag{9}$$

and define the coherent and the chaotic parts as

$$A_{ij,\rm co}=(1-p_{\rm dir})\sqrt{A_{ii}^{\pi}A_{jj}^{\pi}} \tag{10}$$

and

$$A_{ij,\rm ch}=p_{\rm dir}A_{ij}^{\pi}+\sum_{{\rm res}=\rho,\omega,\eta,\ldots} A_{ij}^{\rm res} , \tag{11}$$

respectively. The Bose-Einstein correlation function then takes the form

$$C_2(\mathbf{p}_1,\mathbf{p}_2)=1+\frac{|A_{12,\rm ch}|^2+2{\rm Re}(A_{12,\rm ch}\sqrt{A_{11,\rm co}A_{22,\rm co}})}{A_{11}A_{22}} . \tag{12}$$

We introduce the notation

$$d_{12}\equiv\frac{A_{12,\rm ch}}{\sqrt{A_{11,\rm ch}A_{22,\rm ch}}},\quad p_{\rm eff}\equiv\frac{A_{ii,\rm ch}}{A_{ii,\rm ch}+A_{ii,\rm co}} \tag{13}$$

(we do not consider the explicit momentum dependence of $p_{\rm eff}$). The index eff refers to the fact that the chaoticity arises as a consequence both of the intrinsic chaoticity of direct pions and of resonances [cf. Eq. (17) below]. One then obtains

$$C_2(\mathbf{p}_1,\mathbf{p}_2)=1+2p_{\rm eff}(1-p_{\rm eff}){\rm Re}d_{12}+p_{\rm eff}^2|d_{12}|^2 , \tag{14}$$

which leads, e.g., to two exponentials in C_2, related in a definite way, if one chooses for d_{12} an exponential parametrization (cf. also [22]).

The sensitivity of the correlation function on the chaoticity parameter $p_{\rm dir}$ will be illustrated in Sec. III D below, where the case of a high-energy nucleus-nucleus collision is considered.

C. Intercept of the correlation function

When discussing the intercept of the correlation function, one has to bear in mind that all experiments have a finite resolution in the momentum difference q. Consequently, any value of the "intercept" extracted from data is the result of an extrapolation to the point $q=0$. This value depends not only on the resolution in that particular experiment, but also on the functional form that is used to extrapolate (e.g., Gaussian versus exponential). In the remainder of this paper, when we refer to the intercept of the correlation function it is understood that we mean the result of an extrapolation as described above.

In general, each of the following phenomena can influence the intercept: (a) partial coherence, (b) the presence of long-lived resonances, (c) final-state interactions, and (d) particle misidentification. In this section, we will not be concerned with point (c). Usually, the experimental data that are published have already been corrected for Coulomb interactions, and at least part of the strong final-state interactions are taken into account in the treatment of resonance decays.

Let us first consider the effect of partial coherence. One may write the intercept of the Bose-Einstein correlation function [cf. Eq. (14)]

$$I_0=C_2(\mathbf{p},\mathbf{p})=1+2p_{\rm eff}-p_{\rm eff}^2 . \tag{15}$$

Defining the fractions of pions produced directly (chaotically and coherently) and from resonances,

$$f_{\rm ch}^{\rm dir}=\frac{p_{\rm dir}A_{ii}^{\pi}}{A_{ii}},\quad f_{\rm co}^{\rm dir}=\frac{(1-p_{\rm dir})A_{ii}^{\pi}}{A_{ii}} ;$$
$$f^{\rm res}=\frac{\sum_{{\rm res}=\rho,\omega,\eta,\ldots}A_{ii}^{\rm res}}{A_{ii}} , \tag{16}$$

with

$$f_{\rm ch}^{\rm dir}+f_{\rm co}^{\rm dir}+f^{\rm res}=1\ ,$$

the effective chaoticity $p_{\rm eff}$ is related to the true chaoticity parameter $p_{\rm dir}$ as

$$p_{\rm eff}=p_{\rm dir}(1-f^{\rm res})+f^{\rm res}\ . \tag{17}$$

In Fig. 1 the intercept of the correlation function is shown as a function of $p_{\rm dir}$ and $f^{\rm res}$. In order to read off the fraction of *direct* chaotically produced particles, $p_{\rm dir}$, from the intercept of the correlation function, one has to extract the effective chaoticity $p_{\rm eff}$ according to Eq. (15) and then correct for the fraction of pions from resonance decays. Note that $p_{\rm eff}<p_{\rm dir}$. In particular, if a large fraction of pions arise from resonance decays, $p_{\rm eff}\to 1$ and it will take very precise measurements of the two-particle correlation function at small **q** to determine the true chaoticity, $p_{\rm dir}$.

A further complication arises if a fraction of particles is the decay product of long-lived resonances. As the latter decay at large distances from the source, they contribute to the interference peak only at small momentum differences which may be not accessible due to the finite experimental resolution.

Let us introduce the fractions of pions produced by the decay of "short"- and "long"-lived resonances $f_s^{\rm res}$ and $f_l^{\rm res}$, respectively, with $f^{\rm res}=f_s^{\rm res}+f_l^{\rm res}$. Here, the definition of "long lifetime" refers to the resolution of the detector in a given experiment. To be specific, let us assume that the detector can resolve momentum differences $q>q_c$. If the resonances are taken to travel at the speed of light, this would imply that only the contribution of the decays of resonances with widths $\Gamma>q_c$ to the interference peak can be experimentally resolved in that particular experiment. Extrapolation to $q=0$ then leads to the effective intercept

$$I_0=C_2(\mathbf{p},\mathbf{p})=1+2\sqrt{\lambda}p_{\rm eff}(1-p_{\rm eff})+\lambda p_{\rm eff}^2\ , \tag{18}$$

where

$$\sqrt{\lambda}=\frac{f_{\rm ch}^{\rm dir}+f_s^{\rm res}}{f_{\rm ch}^{\rm dir}+f_s^{\rm res}+f_l^{\rm res}}\ . \tag{19}$$

Finally, consider the effect of particle misidentification. For simplicity, we restrict ourselves to the case of two particle species, pions and kaons; the generalization to more than two particle species is straightforward.

In the laboratory frame a certain, momentum-dependent fraction of K^-,$\xi(\mathbf{p})$, will be misidentified as π^-. The observed single inclusion distribution of pions (including the effects of misidentification) is

$$\frac{1}{\sigma}\frac{d\sigma^{\rm mis}}{d^3p}=\frac{1}{\sigma}\frac{d\sigma^{\pi}}{d^3p}+\xi(\mathbf{p})\frac{1}{\sigma}\frac{d\sigma^{K}}{d^3p}\ , \tag{20}$$

where the superscripts π and K in the terms on the right-hand side (RHS) refer to the "true" single inclusive distributions of pions and kaons, respectively. Similarly the observed $\pi^-\pi^-$ correlation function (including the effects of misidentification) becomes

$$C_2^{\rm mis}(\mathbf{p}_1,\mathbf{p}_2)=1+c_\pi(\mathbf{p}_1)c_\pi(\mathbf{p}_2)[C_2^{\pi\pi}(\mathbf{p}_1,\mathbf{p}_2)-1] +c_K(\mathbf{p}_1)c_K(\mathbf{p}_2)[C_2^{KK}(\mathbf{p}_1,\mathbf{p}_2)-1]\ , \tag{21}$$

where

$$c_\pi(\mathbf{p})=\frac{\frac{1}{\sigma}\frac{d\sigma^{\pi}}{d^3p}}{\frac{1}{\sigma}\frac{d\sigma^{\pi}}{d^3p}+\xi(\mathbf{p})\frac{1}{\sigma}\frac{d\sigma^{K}}{d^3p}} \tag{22}$$

and $c_K(\mathbf{p})=1-c_\pi(\mathbf{p})$. The above results are written for the laboratory frame, where pions and kaons of the same three-momentum **p** are misidentified. The transformation to arbitrary frames is straightforward.

All three effects (partial coherence, decay of long-lived resonances, and particle misidentification) are illustrated in Sec. III below, where the formalism is applied to the description of particle production in ultrarelativistic nuclear collisions.

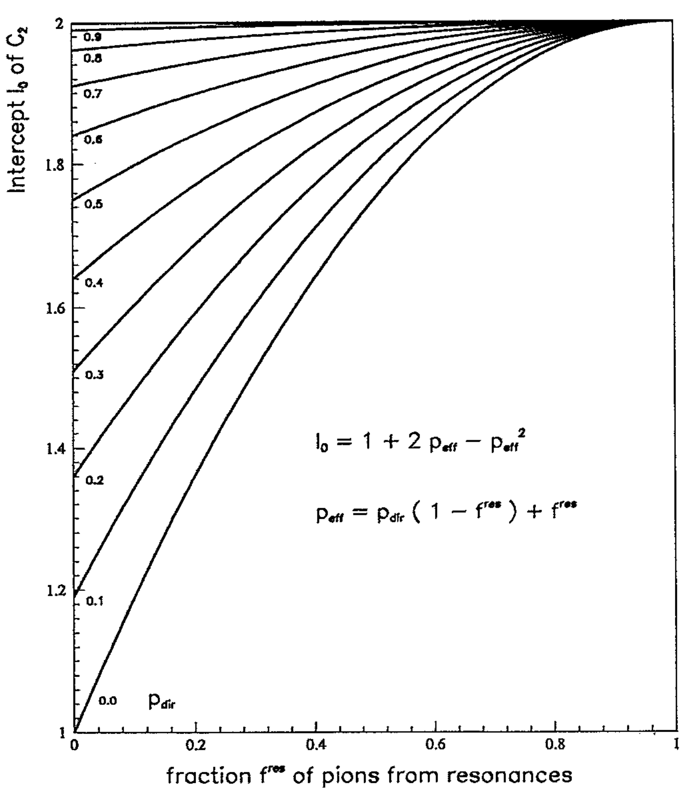

FIG. 1. Intercept of the two-particle correlation function in the presence of coherence and resonances.

III. APPLICATION TO ULTRARELATIVISTIC NUCLEUS-NUCLEUS COLLISIONS

In this section, the formalism presented in the preceding section is applied to describe Bose-Einstein correlations of pions and kaons produced in nuclear collisions at energies reached at the CERN Super Proton Synchrotron (SPS) in the framework of relativistic hydrodynamics. To be specific, we shall consider the reaction $S+S$ at $200A$

TABLE I. Initial fireball quantities describing the experimental $S+S$ data.

Free parameters of the model	
Fraction K_L of thermal energy in Landau volume	42.6%
Longitudinal extension Δ of Landau volume	0.6 fm
Rapidity y_Δ at edge of Landau volume	0.9
Parameter y_m of initial baryon y distribution	0.82
Parameter σ of initial baryon y distribution	0.4

GeV. The principal reason for this choice is that information on baryonic rapidity and transverse momentum distributions is necessary in order to constrain the initial conditions of the hydrodynamic system, and these distributions have so far only been measured for $S+S$ (see Table I).

A. Hydrodynamical model

We consider a hydrodynamic scenario which has been proposed in Ref. [23]. That is to say, we determine the source distribution $g(x,k)$ from a full three-dimensional solution of the relativistic hydrodynamic equations which describes the coupled longitudinal and transverse expansion of the hot and dense matter. The equation of state we use is based on a parametrization of lattice data and exhibits a first-order phase transition. The initial conditions are somewhere between the assumptions of Landau [24] and Bjorken [25]: While about half of the thermal energy is concentrated in a central region of width $\Delta z \sim 1$ fm, the fluid is described by an initial longitudinal velocity field, and the initial baryon number distribution peaks at rapidities $y \sim \pm 0.8$. In the present paper we introduce a minor modification of the formalism of [23] by assuming a more realistic shape for the energy and baryon number initial distribution. This leads to slight changes of the parameters (for details, cf. Appendix B).

B. Bose-Einstein correlation functions

For central collisions, azimuthal symmetry implies that the two-particle correlation function depends only on five variables: $k_\parallel$, $k_\perp$, $q_\parallel$, q_{side}, and q_{out}, where the labels $\parallel$ and $\perp$ refer to the components of $\mathbf{q}$ and $\mathbf{k}$ parallel and transverse to the beam direction, and out and side denote the components of $\mathbf{q}_\perp$ parallel and orthogonal to $\mathbf{k}_\perp$, respectively.

For simplicity, let us begin by assuming a completely

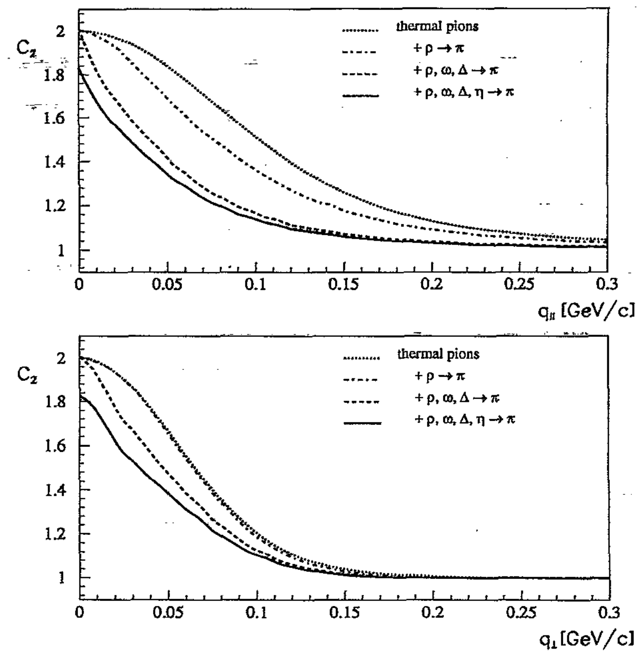

FIG. 2. Bose-Einstein correlation functions of negatively charged pions, in the longitudinal and transverse directions. The separate contributions from resonance are successively added to the correlation function of direct (thermal) π^- (dotted line). The solid line describes the correlation function of all π^-.

chaotic source; the effects of partial coherence will be discussed in Sec. III D below. The resonances which contribute significantly to the production of pions and kaons, and their dominant decay modes, are listed in Table II. Figure 2 illustrates the effect of successively adding the contributions from ρ, ω, Δ, and η decays to the BEC functions of directly produced (thermal) π^- (dotted lines), in longitudinal and in transverse directions. The width of the correlation progressively decreases as the decays of resonances with longer lifetimes are taken into account, and the correlation loses its Gaussian shape. An interesting effect occurs when the η decays are included (solid lines). As a result of the small width of the η of $\sim 10^{-3}$ MeV, the contribution of η decays to the $\pi^-\pi^-$ interference peak cannot be observed experimentally due to the limited detector resolution in the momentum difference $\mathbf{q}$. Thus, one finds an *apparent* drop of the intercept of the correlation function to a value below 2 [of course, the "true" intercept remains equal to 2; cf. Eq. (2)].

TABLE II. Resonance decays included in the calculation of Bose-Einstein correlation functions (except K^*).

Name	m (MeV)	Γ (MeV)	τ (fm/c)	Decays
η	548	1.2×10^{-3}	1.64×10^5	3π (56%), 2γ (39%), $\pi^+\pi^-\gamma$ (5%)
ρ	770	151	1.3	2π
ω	783	8.4	23.4	3π (90%), $\pi^0\gamma$ (8.5%), $\pi^+\pi^-$ (1.5%)
K^*	892	50	3.94	$K\pi$
Δ	1232	115	1.21	$N\pi$

C. Pion versus kaon interferometry

Current experiments at the SPS and the BNL Alternating Gradient Synchrotron (AGS) [26,27] make it possible for the first time to simultaneously measure Bose-Einstein correlations (BEC's) of both pions and kaons produced in relativistic nucleus-nucleus collisions. Ideally, a comparison of the results of pion and kaon interferometry should lead to conclusions concerning possible differences in the space-time regions where these particles decouple from the hot and dense matter. For instance, it has been proposed that kaons may decouple (freeze out) at earlier times and higher temperatures than pions [28]. Indeed, preliminary results tend to indicate that the effective longitudinal and transverse source radii extracted from $\pi\pi$ correlations are significantly larger than those obtained from KK correlations [27]. However, as we have seen in Fig. 2 above, the BEC's of pions are strongly distorted by the contributions from resonance decay. It was pointed out in Ref. [3] in a study based on the Lund string model that such distortions are not present for the BEC's of kaons, and that consequently for the effective transverse radii one expects $R_\perp(K^\pm) < R_\perp(\pi^\pm)$, even in the absence of any difference in the freeze-out geometry of *directly produced* pions and kaons. We confirm these conclusions within the hydrodynamical approach and find furthermore that this effect is even more pronounced if one considers longitudinal rather than transverse radii. Furthermore, the interplay between coherence and resonance production is not considered in [3] (cf. Sec. III D). Note that there are some striking differences also in the resonance production cross sections between [3] and the present paper. We get, e.g., about 50% pions from resonances while in [3] this percentage is 80%. The K^* contributes in [3] with 50% to K^- production while in our treatment only with 5%. In Ref. [3] η and η' contributions are neglected while we consider them and find that they have a strong effect on the intercept. Thus, for a comparison of the results of pion versus kaon interferometry to be meaningful, it is extremely important to understand exactly where in phase space the resonance decays contribute, and how they affect the two-particle correlation functions. Figure 3 shows the rapidity and transverse momentum dependence of the contributions to π^- production of the decays of the resonances listed in Table II.

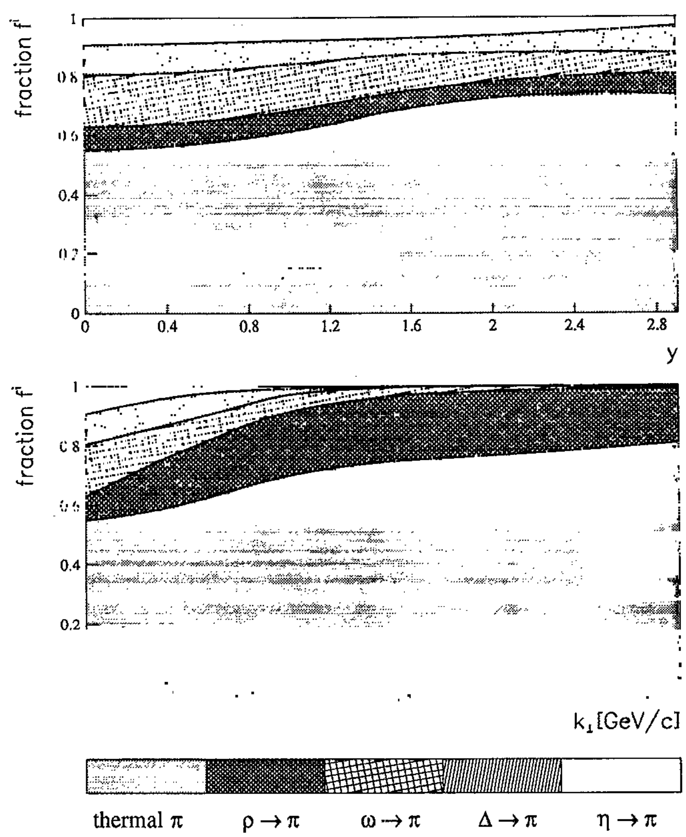

FIG. 3. Rapidity and transverse momentum dependence of the fractions of π^- produced thermally and through vari us resonance decays, respectively.

In Fig. 4, BEC functions of π^- (solid lines) and of K^- (dashed lines) are compared, at $k_\perp=0$ and $k_\perp=1$ GeV/c, respectively. The dotted lines correspond to the BEC function of thermally produced π^-. It can be seen that the distortion due to the decay contributions from long-lived resonances disappears only at large $k_\perp$.

A characteristic property of particle production from an expanding source is a correlation between the space-time point where a particle is emitted and its energy momentum [17]. As a consequence, the effective source radii extracted from Bose-Einstein correlation functions, i.e., the inverse widths of the correlation functions, show a characteristic dependence on the average momentum of the pair, k. In order to extract effective source radii, we fit our results to the Gaussian form used by experimentalists:

$$C_2(\mathbf{p}_1,\mathbf{p}_2)=1+\lambda\exp\left[-\frac{1}{2}\underbrace{(q_\parallel^2R_\parallel^2+q_{\rm side}^2R_{\rm side}^2+q_{\rm out}^2R_{\rm out}^2)}_{\equiv\Sigma(qR)^2}\right]. \quad (23)$$

The parameter λ takes into account the effective reduction of the intercept due to the contribution from η decays. It should be emphasized that here λ does not represent the effect of coherence. Figure 5 shows the effective radii $R_\parallel$, $R_{\rm side}$, and $R_{\rm out}$ as functions of rapidity and transverse momentum of the pair, both for $\pi^-\pi^-$ (solid lines) and for K^-K^- pairs (dashed lines). For comparison, we have also included the curves for thermally produced pions (dotted lines). The qualitative dependence on y_k and $k_\perp$ has been discussed extensively

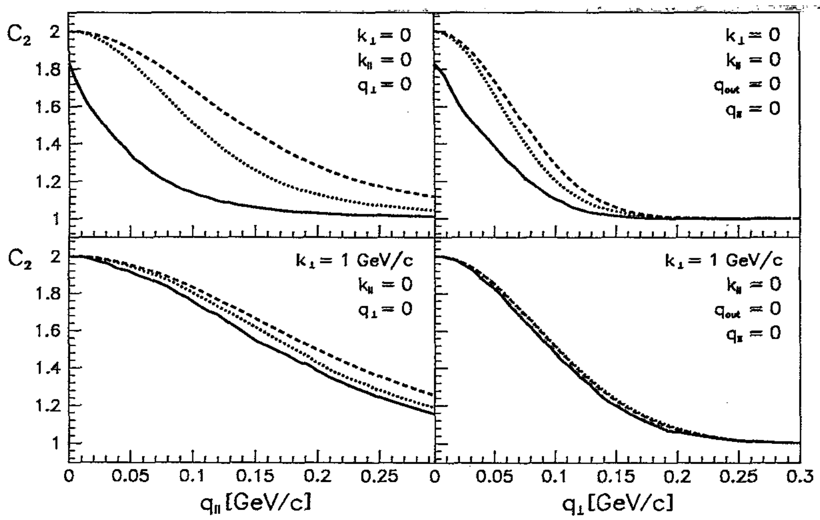

FIG. 4. Correlation functions of all π^- (solid lines), thermal π^- (dotted lines), and all K^- (dashed lines), for $k_\perp=0$ and for $k_\perp=1$ GeV/c.

in Ref. [16]. Here, we would like to emphasize that the effective longitudinal radii extracted from $\pi^-\pi^-$ correlations are considerably larger than those obtained from K^-K^- correlations. In the central region, the two values for $R_\parallel$ differ by a factor of ~ 2. For the transverse radii, the factor is ~ 1.3. A comparison between results for K^- and thermal π^- shows that part of this effect can be accounted for by kinematics [the pion mass being smaller than the kaon mass; see also Eq. (24) below]. Nevertheless, the large difference between the widths of pion and kaon correlation functions is mainly due to the fact that pion correlations are strongly affected by resonance decays, which is not the case for the kaon correlations. In our hydrodynamic scenario, about 50% of the pions in the central rapidity region are the decay products of resonances [23], while less than 10% of the kaons are created in resonance decays ($K^* \rightarrow K\pi$ dominates, contributing about 5%).

Note that here we have been conservative in assuming that thermal pions and kaons are produced on the same freeze-out hypersurface. If kaons decouple from the excited matter earlier than pions, the difference in $R_\parallel$ is even more dramatic than what we have shown. Conversely, if one intends to obtain information about a pos-

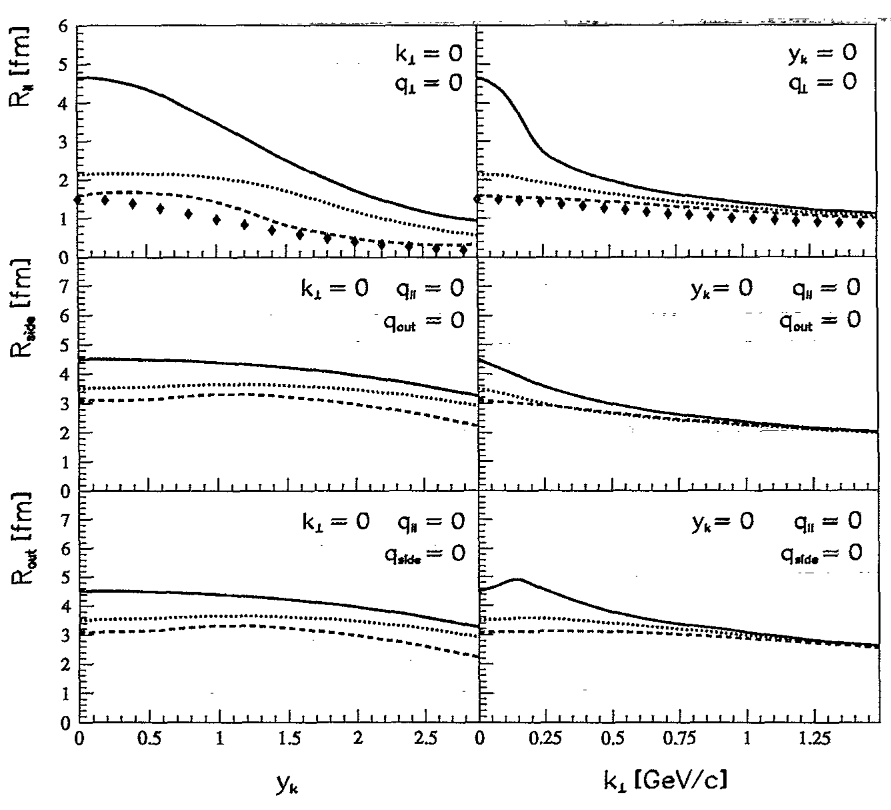

FIG. 5. Dependence of the longitudinal and transverse radii extracted from Bose-Einstein correlation functions on the rapidity y_k and average momentum $k_\perp$ of the pair, for all π^- (solid lines), thermal π^- (dotted lines), and all K^- (dashed lines).

sible difference in freeze-out geometries of pions and kaons by comparing the widths of $\pi\pi$ and KK correlation functions, one has to restrict oneself to a region in momentum space where resonances do not contribute (unless one is able to identify thermally produced pions). Our results indicate that only at very large transverse momenta $k_\perp \geq 0.8$ GeV/c the contributions from resonance decays can be neglected (cf. Fig. 5).

In Ref. [16], where the effect of transverse flow on the Bose-Einstein correlations of thermally produced pions was studied, we observed that the effective longitudinal radii agreed remarkably well with an approximate analytic expression which was derived for a one-dimensional (purely longitudinal) expansion in Refs. [29,30] (cf. also [31]):

$$R_\parallel = \left[\frac{2T_f}{m_\perp}\right]^{1/2} \frac{\tau_0}{\cosh(y_k)}, \tag{24}$$

where $m_\perp = (m_{\perp 1} + m_{\perp 2})/2$ is the average transverse mass of the two particles, T_f is the freeze-out temperature, and $\tau_0 = (\partial u_\parallel / \partial z)^{-1}$ is the inverse gradient of the longitudinal component of the four-velocity in the center (at $z=0$). As was argued in [16], for large freeze-out times the solution at $z=0$ is close to the scaling solution, and consequently, for a one-dimensional expansion $\tau_0 \approx t_f(z=0)$, where $t_f(z)$ is the freeze-out time at longitudinal position z. For a three-dimensional expansion, $t_f = t_f(z, r_\perp)$, and one has $\tau_0 \approx \langle t_f(z=0, r_\perp) \rangle$, where the average is taken over the radial coordinate.

We have checked that this result remains valid for the K^-K^- correlations in the more realistic scenario considered here. The symbols $\Diamond$ in Fig. 5 were obtained by substituting the value $\langle t_f(z=0, r_\perp) \rangle = 2$ fm/c for the average lifetime of the system (calculated directly from hydrodynamics by averaging over the hypersurface) into Eq. (24), with $T_f = 0.139$ GeV. The good agreement suggests that it is possible to determine the lifetime of the source from a measurement of the kaon correlation functions in the longitudinal direction.

A comprehensive analysis of Bose-Einstein correlations of both pions and kaons will be necessary to obtain information about the space-time aspects of multiparticle production in high-energy nuclear collisions. In particular, data ought to be taken in different rapidity and transverse momentum bins of the pair. An interesting issue which has not been discussed here concerns a possible separation of strange and anti-strange matter [28,32,33], which can be investigated by comparing correlations of K^+K^+, K^-K^-, and K_SK_S pairs (see Ref. [34] and references therein).

D. Partial coherence

Figure 6 shows the $\pi^-\pi^-$ correlation functions in the presence of partial coherence. In order to extract effective radii from Bose-Einstein correlation functions in the presence of partial coherence, Eq. (23) must be replaced by the more general form

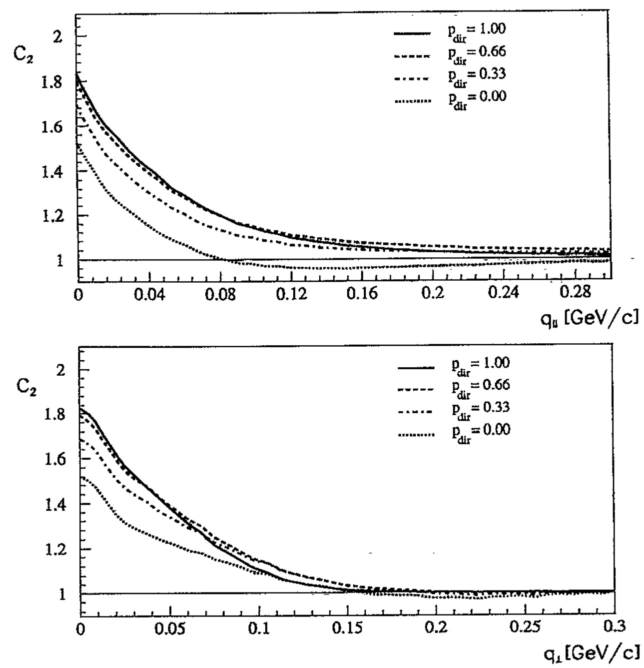

FIG. 6. $\pi^-\pi^-$ Bose-Einstein correlation functions in the presence of partial coherence.

$$C_2(\mathbf{p}_1, \mathbf{p}_2) = 1 + \lambda p_{\rm eff}^2 \exp\left[-\frac{1}{2}\sum (qR)^2\right] + \sqrt{\lambda}\, 2p_{\rm eff}(1-p_{\rm eff}) \exp\left[-\frac{1}{4}\sum (qR)^2\right]. \tag{25}$$

The effective radii obtained by fitting our results with Eq. (25) are listed in Table III.

E. Particle misidentification

Finally, let us consider the effects of particle misidentification on the two-particle correlation function. As in Sec. II C, for illustration we shall take into account the misidentification of K^- as π^- in the *laboratory frame.* In order to estimate the maximum effect, we assume that *all* K^- are identified as π^-, $\xi(\mathbf{p}) \equiv 1$. In Fig. 7, the resultant correlation function (dashed line) in the c.m. frame is compared to the true $\pi^-\pi^-$ correlation function (solid line). Clearly, for this case the effect on the shape of the correlation function is negligible.

TABLE III. $p_{\rm eff}$, λ, intercept I_0, and effective radii as a function of $p_{\rm dir}$ in Fig. 6.

$p_{\rm dir}$	$p_{\rm eff}$	λ	I_0	$R_\parallel$ (fm)	$R_\perp$ (fm)
1.00	1.00	0.82	1.82	4.60	4.51
0.66	0.82	0.79	1.79	5.08	4.61
0.33	0.63	0.73	1.69	6.61	4.91
0.00	0.45	0.63	1.52	11.01	5.89

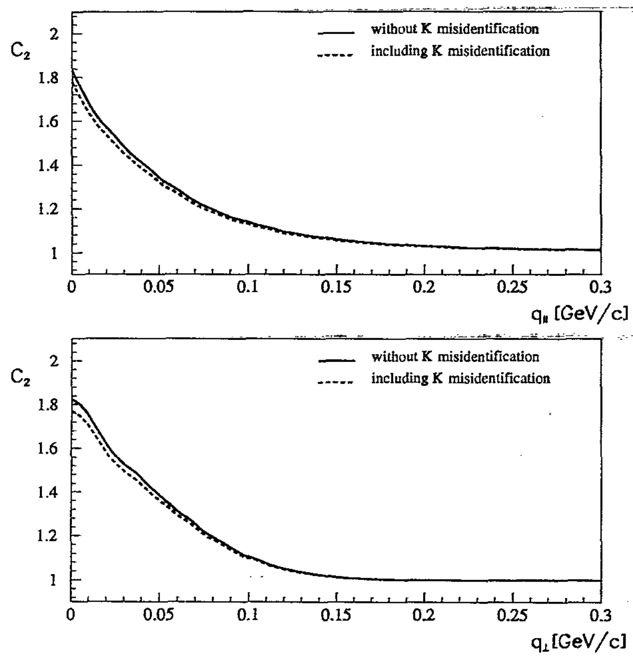

FIG. 7. $\pi^-\pi^-$ correlation functions, with (dashed lines) and without (solid lines) the effect of misidentification of kaons for pions.

IV. CONCLUSIONS

In a consistent treatment of single and double inclusive cross sections for identical pions via a realistic hydrodynamical model, resonances play a major role leading to an increase of effective radii of sources. Effective longitudinal radii are more sensitive to the presence of resonances than transverse ones.

The contribution from the decay of long-lived resonances also affects strongly the intercept of the correlation function $C_2(q)$ at $q=0$. Particle misidentification has similar effects; however, the contribution from misidentification of kaons as pions has been shown to be small. These effects affect any conclusions about the presence of coherence, based on the value of the intercept. This is much less the case for the two-exponential structure of the correlation function which is a specific consequence of partial coherence [cf. Eq. (14)].

For kaons the effect of resonances is much less important. The effective radii obtained from kaon interferometry are therefore much smaller than those of pions and information about coherence derived from the intercept is more reliable. The hydrodynamical treatment provides a consistent description of single inclusive and double inclusive cross sections for secondaries. Predictions for BEC's in $S+S$ reactions made in the present paper if compared with experimental data (unavailable so far) could constitute a sensitive test of the equation of state and the source formalism for Bose-Einstein correlations.

ACKNOWLEDGMENTS

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APPENDIX A: EVALUATION OF THE CORRELATOR INTEGRALS

For directly produced pions we get immediately, from insertion of Eq. (6) into Eq. (8),

$$A_{ij}^{\pi}=\frac{1}{(2\pi)^3}\int_\Sigma \frac{k^\mu d\sigma_\mu(x'_\mu)}{\exp\left[\dfrac{k^\mu u_\mu(x'_\mu)}{T_f(x'_\mu)}\right]-1}e^{iq^\mu x'_\mu}. \tag{A1}$$

Especially for the single inclusive we obtain

$$A_{ii}^{\pi}=\frac{1}{(2\pi)^3}\int_\Sigma \frac{p_i^\mu d\sigma_\mu(x'_\mu)}{\exp\left[\dfrac{p_i^\mu u_\mu(x'_\mu)}{T_f(x'_\mu)}\right]-1}. \tag{A2}$$

For the $A_{ij}^{\rm res}$ we can perform several integrations. Insertion of Eq. (4) into Eq. (9) and integration over x gives

$$A_{ij}^{\rm res}=\int \frac{d^3p^*}{E^*}d^4x^*d\tau\,\Gamma\exp(-\Gamma\tau)\Phi_{\text{res}\to\pi}(p^{*\mu},k^\mu)$$
$$\times g_{\rm res}^{\rm dir}(x_\mu^*,p^{*\mu})\exp\left[iq^\mu\left[x_\mu^*+\frac{\tau}{m^*}p_\mu^*\right]\right]. \tag{A3}$$

With

$$\int_0^\infty d\tau\,\Gamma\exp(-\Gamma\tau)\exp\left[i\frac{\tau}{m^*}p_\mu^*q^\mu\right]=\frac{1}{1+i\dfrac{p^{*\mu}q_\mu}{m^*\Gamma}} \tag{A4}$$

and insertion of Eqs. (6) and (5) into Eq. (A3), one obtains, after integration over x^* in the case of two-particle resonance decay,

$$A_{ij}^{\rm res}=\frac{b}{4\pi p_0}\int\frac{d^3p^*}{E^*}\delta\left[E_0-\frac{p_\mu^*k^\mu}{m^*}\right]$$
$$\times\frac{2J+1}{(2\pi)^3}\int_\Sigma\frac{\exp(iq^\mu x'_\mu)}{\left[1+i\dfrac{p^{*\mu}q_\mu}{m^*\Gamma}\right]}\frac{p^{*\mu}d\sigma_\mu(x'_\mu)}{\exp\left[\dfrac{p^{*\mu}u_\mu(x'_\mu)-B\mu_B(x'_\mu)-S\mu_S(x'_\mu)}{T_f(x'_\mu)}\right]\pm1}. \tag{A5}$$

Using the definitions

$$p^{*\mu}=(m_\perp^*\cosh y^*,\ m_\perp^*\sinh y^*, p_\perp^*\cos\varphi^*, p_\perp^*\sin\varphi^*)\ ,$$

$$k^\mu=(m_{\perp k}\cosh y_k, m_{\perp k}\sinh y_k, k_\perp\cos\varphi_k, k_\perp\sin\varphi_k)\ ,$$

and

$$\frac{d^3p^*}{E^*}=\frac{1}{2}dy^*dp_\perp^{*2}d\varphi^*\ ,$$

one finally gets, after eliminating the δ function by integrating Eq. (A5) with respect to φ^*,

$$A_{ij}^{\rm res}=\frac{bm^*}{8\pi p_0}\int_{y^*_{(-)}}^{y^*_{(+)}}\int_{p_\perp^{*2(-)}}^{p_\perp^{*2(+)}}\frac{dp_\perp^{*2}dy^*}{\sqrt{(p_\perp^*k_\perp)^2-[E_0m^*-m_{\perp k}m_\perp^*\cosh(y_k-y^*)]^2}}$$
$$\times\sum_{n=1}^{2}\left[\frac{2J+1}{(2\pi)^3}\int_\Sigma\frac{\exp(iq^\mu x'_\mu)}{\left[1+i\frac{p^{*\mu}q_\mu}{m^*\Gamma}\right]}\frac{p^{*\mu}d\sigma_\mu(x'_\mu)}{\exp\left[\frac{p^{*\mu}u_\mu(x'_\mu)-B\mu_B(x'_\mu)-S\mu_S(x'_\mu)}{T_f(x'_\mu)}\right]\pm1}\right]_{\varphi_n^*}\ , \tag{A6}$$

where

$$\varphi_1^*=\varphi_k+\arccos\left[\frac{1}{p_\perp^*k_\perp}[m_\perp^*m_{\perp k}\cosh(y_k-y^*)-E_0m^*]\right]\ ,$$

$$\varphi_2^*=2\pi-\varphi_1^*\ ,$$

$$p_\perp^{*2(\pm)}=m_\perp^{*2(\pm)}-m^{*2}\ ,$$

$$m_\perp^{*(\pm)}=\frac{m^*\left[E_0m_{\perp k}\cosh(y_k-y^*)\pm k_\perp\sqrt{E_0^2+k_\perp^2-m_{\perp k}^2\cosh^2(y_k-y^*)}\right]}{m_{\perp k}^2\cosh^2(y_k-y^*)-k_\perp^2}\ ,$$

$$y^*_{(\pm)}=y_k\pm\ln\left[\frac{\sqrt{E_0^2+k_\perp^2}+p_0}{m_{\perp k}}\right]\ .$$

For the contribution to the single inclusive we obtain the well-known result

$$A_{ii}^{\rm res}=\frac{bm^*}{4\pi p_0}\int_{y^*_{(-)}}^{y^*_{(+)}}\int_{p_\perp^{*2(-)}}^{p_\perp^{*2(+)}}\frac{dp_\perp^{*2}dy^*}{\sqrt{(p_\perp^*p_{\perp i})^2-[E_0m^*-m_{\perp i}m_\perp^*\cosh(y_i-y^*)]^2}}$$
$$\times\frac{2J+1}{(2\pi)^3}\int_\Sigma\frac{p^{*\mu}d\sigma_\mu(x'_\mu)}{\exp\left[\frac{p^{*\mu}u_\mu(x'_\mu)-B\mu_B(x'_\mu)-S\mu_S(x'_\mu)}{T_f(x'_\mu)}\right]\pm1}\ . \tag{A7}$$

The expressions in the case of three-body decays can be reduced to those corresponding to two-particle decay [18].

APPENDIX B: SHAPE OF THE INITIAL ENERGY DENSITY AND BARYON NUMBER DISTRIBUTIONS

In [23], it was assumed that the initial energy density and baryon number distributions $\epsilon(z,r_\perp)$ and $n_B(z,r_\perp)$ have sharp surfaces in the transverse direction; i.e., they were taken to be proportional to $\Theta(R-r_\perp)$ (where z and $r_\perp$ are the coordinates in the longitudinal and transverse directions). Here, we generalize these initial conditions of Ref. [23] by taking into account the smearing out of initial distributions in the transverse direction as follows. Let $\rho(r)$ denote the nuclear density distribution ($r^2\equiv r_\perp^2+z^2$). The energy and baryon number deposited at coordinates $(r_\perp,\phi)$ is proportional to

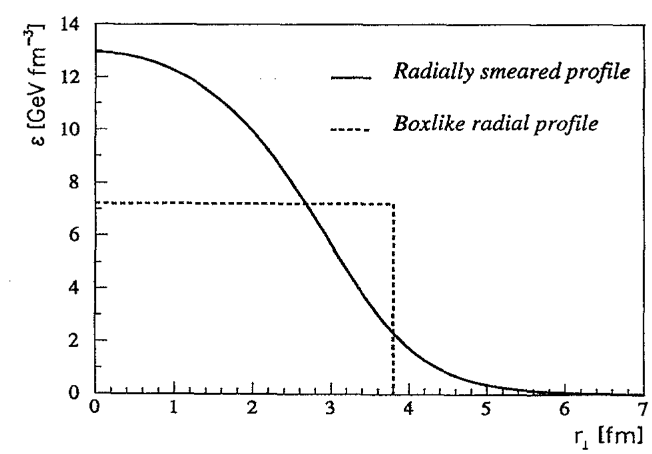

FIG. 8. Energy density profile in the radial (transverse) direction.

$$\bar{\rho}(r_\perp) \equiv \frac{\int_{-\infty}^{\infty} dz\, \rho(\sqrt{z^2+r_\perp^2})}{\int d^2 r_\perp \int_{-\infty}^{\infty} dz\, \rho(\sqrt{z^2+r_\perp^2})} . \tag{B1}$$

Thus, we write

$$\begin{aligned} \epsilon(z,r_\perp) &\sim \bar{\epsilon}(z)\bar{\rho}(r_\perp) , \\ n_B(z,r_\perp) &\sim \bar{n}_B(z)\bar{\rho}(r_\perp) , \end{aligned} \tag{B2}$$

where $\bar{\epsilon}(z)$ and $\bar{n}_B(z)$ are parametrized as in [23]. For the density distribution of the sulfur nucleus, we take a Woods-Saxon parametrization (cf. for example, [35]). The parameters of the initial energy density, baryon number, and velocity distribution were obtained by fitting the experimentally observed [36] rapidity and transverse momentum distributions both of negative particles and of protons; their values are listed in Table I. The resultant profile of the initial energy density distribution at $z=0$ in the transverse direction is plotted in Fig. 8. For comparison, we have also included the boxlike profile which was assumed in Ref. [23].

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